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SOME CRITICAL REMARKS ON ANALYTICAL REALISM

I

IN 1903 appeared Mr. Bertrand Russell's "Principles of Mathematics," a book which has attracted widespread interest. It had the merit of discussing in a fairly accurate and sometimes witty manner the fundamental mathematical disciplines, such as geometry, mechanics, arithmetic, and transfinite assemblages, and of attempting to relate these subjects to a system of philosophy, namely, the "pluralism" of Mr. G. E. Moore, "which regards the world, both that of existents and that of entities, as composed of an infinite number of mutually independent entities, with relations which are ultimate and not reducible to adjectives of their terms or of the whole which these compose."[1] Russell's treatment has been called *neo-realism*.[2] The mathematical advantage of this philosophical position, Russel maintains, is that, unlike most current philosophies, it allows mathematics to be true[3] in a sense which he has frequently sought to explain in various articles and which need not be dwelt upon further here.[4]

As to his method Russell says: "Our method will be one of analysis, and our problem may be called philosophical in the sense that we seek to pass from the complex to the simple, from the demonstrable to its indemonstrable premisses." Also Russell does not distinguish[5] between inference and deduction; induction appears[6] to him as "either disguised deduction or a mere method of making plausible guesses." It is natural enough that he should find the relation of

[1] "Principles of Mathematics," page viii.

[2] H. Dufumier, *Revue de Metaphysique et de Morale*, Vol. 17 (1909), page 620.

[3] *Loc. cit.*, page viii.

[4] *Cf.*, for instance, "The Problems of Philosophy," London; also Russell's "Philosophical Essays," New York, 1910.

[5] "The Principles of Mathematics," § 1.

[6] *Cf.* also *Monist*, Vol. 23 (1913), pages 489–490.

whole and part (which had been previously discussed by De Morgan) fundamental for his position; indeed, he says that for the comprehension of analysis it is necessary to investigate this notion.[7]

As might be expected from Russell's realism, his relational position is the so-called external one, which is opposed to the internal theories favored by idealists. An external relation is described as one implying no complexity in either of the related terms.[8] According to him[9] there exist external relations because asymmetrical relations are involved in Number, Quantity, Order, Space, Time and Motion, and it is impossible for him to explain asymmetrical relations on either of the usual theories of relation, *i. e.*, the monistic and the monadistic.[10] Concerning the external treatment of relations, it is important to recognize a contention made by Russell which, I think, has been supported[11] by Couturat: Russell implies that his discussion may serve as an engine of discovery in actual mathematics.[12]

Russell was able to carry out his discussion, as a whole, largely because he could avail himself of Peano's symbolic formulary of mathematics, whose principles he freely incorporated into his book and from whose mathematical content he drew much of his inspiration.[13]

II

In recent years, a group of young philosophers, "six realists" as they called themselves, has been engaged in controversy, notably with Dewey.[14] Presumably as an outcome of their activities they published a book last year, "The New Realism," consisting of six essays. One of these, "A Defense of Analysis," by Professor E. G. Spaulding, of Princeton, deals to a considerable extent with mathematics and will receive critical consideration in the present paper. Spaulding's purpose is to defend the general realistic interpretation of whole and part, to classify wholes into certain types, and to show that the analysis of each kind of whole does not lead to falsification.[15] The kinds of wholes Spaulding discusses are four:

I. Collections in numerical conjunction.

II. Classes formed or composed of parts which are not classes, but

[7] *Loc. cit.*, page 11, note; *cf.* A. T. Shearman, "The Development of Symbolic Logic," pages 203–205.

[8] *Loc. cit.*, chapter XVI.

[9] *Loc. cit.*, page 224, paragraph 1.

[10] *Loc. cit.*, § 216.

[11] *Cf. Monist*, Vol. 22 (1912), page 524.

[12] *Loc. cit.*, page 24, § 27.

[13] See Russell's own statements, *loc. cit.*, page 26, § 31, and elsewhere.

[14] This JOURNAL, Vol. VIII. (1911).

[15] "The New Realism," pages 155, 157, 168.

which may be either organic wholes or individuals or simples or collections.

III. Classes formed or composed of subordinate classes.

IV. Unities or organic wholes.

Each of these "wholes" or rather specific instances of the latter, Spaulding examines in turn, in order to establish the thesis that analysis is the discovery of the parts of a whole and the organizing relations which these parts sustain to each other.[16]

III

1. It is not my intention to consider Spaulding's essay point by point in detail and in the order of his article. I shall relate myself to crucial philosophic statements and then consider such mathematical errors as are typical.

A comparison of Spaulding's article with Russell's "Principles of Mathematics" shows at once that he has tried to carry out the Russell programme. Indeed, he has almost literally followed Russell in many instances, especially as to mathematics, and, when he deviates from Russell, frequently falls into errors of a very obvious nature.

One of the first points that Spaulding[17] makes is that "all the attacks on analysis are made by methods which themselves involve analysis or are analytical." In testing the effectiveness of any philosophical system, we have the right to investigate its utility (1) with reference to known contents, (2) with reference to unsolved problems. As to the second consideration, we are asked to take the word of the realists that their analysis is a means of discovery, but for tangible evidence they, in effect, refer us to analyses of known contents. Not being creative mathematicians themselves, it would be well for the realists to note what means of discovery eminent mathematicians employ to solve problems in research; but then they would find that the logical position of these investigators is very different from their analytical realism.[18] When in the presence of an unsolved problem even Russell abandons his method of analysis and becomes—as an examination will show—typically inductive; thus illustrating Dewey's remark[19] that a universal seems necessarily as-

[16] *Loc. cit.*, pages 158, 161, 168. It is hardly necessary to recall Aristotle's well-known comparison of a "whole" with an "organism."

[17] *Loc. cit.*, page 160.

[18] *Cf.* H. Poincaré *L'Enseignement Mathématique*, Vol. 10 (1908), pages 357–371. This position of Poincaré has been criticized by E. Borel, *Revue du Mois*, Vol. 7 (1909), page 98. Further, G. Cantor has stated his own general position to be Aristotelian realism. See *Zeitschrift für Philosophie*, Vol. 91 (1887), page 86.

[19] *Decennial Pub. of U. of C.*, Ser. 1, Vol. 3, page 122.

sociated with the existence of a problem. But even in the analysis of known contents the realists are unable to free themselves from inductive methods. Peirce has said that the syllogism involves an element of observation,[20] and a similar remark may be made of all deduction and of all analysis. It is also easy to recognize observational elements of Spaulding's discussion.[21] Spaulding compares the points on a line, the instants of time, and the series of real numbers and finds common properties. Similarly, the so-called "platform" which the six realists have printed as an appendix to their book aims to be a doctrine underlying the six essays which, as they frankly admit, are not in complete agreement. This must be the reply, then, to Spaulding's general criticism of the attacks on analysis.

2. Spaulding's attempted general refutation of the attacks on analysis is incidental to his consideration of specific arguments that analysis is identical with falsification. The instances of attacks upon analysis which Spaulding controverts, hardly do justice to the possibilities. One of these so-called model attacks[22] on analysis runs as follows: "The analysis of space leads to terms which are not spatial; it leads from the extended, the dimensional, to the unextended, the undimensional." Another attack[23] is stated thus: "Space . . . is given empirically by intuition (or some such mode of direct approach) as a unitary continuous whole. But analysis leads to terms or to parts of space which are discrete from one another." Spaulding does not seem to realize[24] that to regard mathematical space as the result of an analysis of a "whole" is itself a false attitude; such a position refers to a preliminary, perceptual space rather than a complete space as a concept. My contention is that the analysis of a "whole" could never induce the mathematical space Spaulding implies in his essay, but only a limited portion of this space. The problem of the genesis of one of those mathematical contents to which the generic name *geometry* is applied is, of course, classic.[25] In the heuristic development of space, as I conceive it, we have first presented to us a perceptual spatial content; in this perceptual "whole," "points" are perhaps spatial magnitudes, "lines" or "rods" are of limited length and have thickness, "planes" or "plates" are of limited area and have thickness, etc.[26] Comparison of such perceptual

[20] *American Journal of Mathematics*, Vol. 7 (1884–1885), page 182; *cf.* G. H. Mead, *Phil. Rev.*, Vol. 9, pages 5–9.

[21] For instance, on page 184 of his essay.

[22] Spaulding, *loc. cit.*, page 186.

[23] Spaulding, *loc. cit.*, page 186.

[24] *Loc. cit.*, pages 169, 187.

[25] *Cf.* Hölder, "Auschauung und Denken in der Geometrie," page 2.

[26] *Cf.* Veronese, "Grundlagen der Geometrie," pages 52–56, 225, 226, etc.

spaces leads to the organization of what may be called a fragmentary conceptual space. By a generalizing, constructive process in which the principle of complete induction plays an important part, this fragmentary space is completed. The completion may be effected in a variety of ways and, indeed, we have at least three geometries, *viz.*, the Euclidean, the Lobatcheffskian, and the Riemannian. Between such completed spaces and the naïve, perceptual contents which approximate the former, Spaulding has not properly discriminated; nor does he seem to understand that the term "space" in mathematics is rather superficial. In fact, "space" is a name applied to many collections of mathematical elements, but its use is a mere matter of convenience, not essential to mathematics.[27] As in the case of space, so in regard to continuity, time, and motion; Spaulding has not properly recognized the conceptual constructive systems as distinguished[28] from the crude percepts which led to them (in part), and these misunderstandings are quite sufficient to throw out any argument based on them. The thesis that Spaulding and the other realists should refute to show that analysis does not lead to falsification may briefly be stated thus:

Let it be granted that there exist infinite[29] *wholes; then it is not possible to analyze such wholes without leading to contradictions.*

Here is an opportunity for the realists to display their analytical skill. It seems that Russell's analysis resulted in an antinomy,[30] which is strangely at variance with his statement[31] that analysis gives us the truth and nothing but the truth.

3. In a future article I hope to discuss the nature of the whole-part relation, its hypothetical and intuitive significances, etc. I wish here merely to refer to that intuitive aspect of a whole which refers to the parts into which the whole has been analyzed. Granted that a whole has been successfully analyzed into parts, these parts represent a specific choice and this choice is intuitive. There must, then, have been a guiding principle associated with the analysis. Let me give an example. In the case of a descriptive three-space there are at least three analyses possible, each leading to an asymmetrical relation, namely, one-dimensional, two-dimensional, and three-dimensional. For each analysis there exists a specific definition of be-

[27] *Cf.* E. J. Wilczynski, *Bull. Am. Math. Soc.*, Vol. 19 (1913), pages 333–334.

[28] *Cf.* J. Royce, "The World and the Individual," Vol. I., pages 526–588.

[29] *Cf.* Spaulding, *loc. cit.*, pages 157, 201; Russell, *loc. cit.*, Chap. XVII.

[30] *Loc. cit.*, §§ 70, 78, 100, 344. This antinomy, by the way, like many others, seems to have as underlying problem the interdependence of object and act, clearly recognized by Plato, "Parmenides," 135, etc.; "Phædo," 73.

[31] *Loc. cit.*, page 141.

tweenness.[32] Russell and Spaulding have taken account of only one analysis and only one definition of betweenness. Now what is to guide us in adopting one analysis rather than another? Russell has touched[33] upon this question:

"It is important to observe that the definition of a space, as of most other entities of a certain complexity, is arbitrary within certain limits. . . . For example, in place of defining the line by a relation between points, it is possible to define the line as a class having a certain relation to a couple of points. In such cases we can only be guided by motives of simplicity."

It would be interesting to know what Russell[34] means by "motives of simplicity," and it is hard to see what test of simplicity of parts there can be other than "satisfactory functioning." This practical test, indeed, enables us to make a choice of the three analyses mentioned above. A three-dimensional analysis of three-space finds its justification in its relevance to the foundations of vector analysis and the application of the latter to mechanics and physics.[35] The geometric example just cited shows that analysis requires intuitive control, and this control must prevent irrelevant analyses or consideration of irrelevant contents. This deficiency suggests that Russell tends towards *scholasticism.* What Green[36] says of the Aristotelian logic is not without application to Russell and Spaulding:

"Thus the Aristotelian or syllogistic logic earns the reproach of consisting in a series of verbal propositions. It represents neither a method of arriving at knowledge nor the system of ideas which constitute the known world . . . but is merely of use in analyzing what is involved in conceded general propositions. . . . Hence its use by the Schoolmen. They did not want a method of arriving at truth nor a theory of what knowledge consists in. . . . As a rule for securing consistency in the interpretation and application of general terms, syllogistic logic has its value."

While, of course, Russell's position, like that of Boole and De Morgan,[37] occupies broader ground than the syllogistic logic, yet I think that the criticism just quoted suggests a fundamental defect of Russell and Spaulding. This view (as to Russell) finds support in

[32] *Amer. Jour. of Math.*, 1909, page 365.

[33] *Loc. cit.*, page 432.

[34] *Cf.* "Principles of Mathematics," page 251, last lines; page 379, paragraph 4.

[35] *Cf. Amer. Jour. Math.*, 1913, pages 37–56.

[36] "Philosophical Essays," Vol. II., page 160.

[37] Both Boole and De Morgan recognized the inadequacy of the syllogistic logic. *Cf.* "Laws of Thought," page 10, and *Camb. Phil. Trans.*, 1864, page 335.

an able survey[38] of Russell's "Principles of Mathematics" by Hausdorff. The latter says:

"A scholastic acuteness which perceives imaginary problems and neglects real difficulties, celebrates in Russell's book orgies of subtlety." Again, he says:

"In Russell's book are two conflicting tendencies, *viz.*, the formalistic, nominalistic, and one opposite to this for which it is difficult to find a name; an *a priori* tendency, realistic in the medieval sense, which would force us to discriminate, in a definite manner, between what is fundamental and what is derived and leads us to hair-splitting decisions in matters which are purely definitional."

To review briefly; the realistic position of Spaulding and Russell is insufficient to account for "wholes," in particular, those of a mathematical nature; it is inadequate, too, in the control of content if we admit that a whole has been successfully analyzed. The test of ultimacy of an analysis into parts must be found in the satisfactory functioning of these parts. For evidence of irrelevant analyses we have only to turn to Russell's book which seems to indicate ignorance on the part of the author of the practical needs of mathematics and logic.

4. The ultimacy of analysis which formed the subject of the preceding section suggests examination of the Russell-Spaulding treatment of asymmetrical relations and their general theory of relation. Spaulding,[39] citing Russell,[40] says that asymmetrical relations are unintelligible on any other theory than that of external relations; and to justify this statement Russell examines[41] the monistic and monadistic theories of relation and concludes, at least to his own satisfaction, that they are inadequate. Russell has given asymmetrical relations great prominence in his book. In every argument, if he has an opportunity, he leads his readers to an asymmetrical relation. The theory of magnitude, when based on transitive, symmetrical relations seems to Russell paradoxical and complicated; asymmetrical relations provide a simple and consistent theory; geometric order is generated by an asymmetrical relation and similarly in regard to time and motion.[42] Now the question may fairly be asked: Are asymmetrical relations indispensable from a practical standpoint? Of asymmetrical relations in general Royce[43] says:

"The contrast between symmetrical and unsymmetrical relations

[38] *Vierteljahrschrift für wiss. Phil. u. Soz.*, Vol. 29 (1905), pages 119–124.

[39] *Cf.* Spaulding, *loc. cit.*, page 176, note.

[40] Russell, *loc. cit.*, § 216.

[41] Russell, *loc. cit.*, §§ 212–215.

[42] *Cf.* Russell, *loc. cit.*, §§ 154–157; 206–207; 441, 446.

[43] *Transactions of the American Mathematical Society*, Vol. 6 (1905), pages 358–359.

seems, to the ordinary view, absolute. Mr. Russell, in his late volume, so treats it. . . . In symbolic logic, however, a symmetrical copula, namely, that of 'inconsistency' or of 'opposition' can be made to accomplish all the work of the ordinary unsymmetrical copula $-<$. In other words, if I have otherwise defined the meaning of 'not,' the statement 'x is inconsistent with not-y' means the same as 'x implies y.' The copula in the former case is symmetrical, in the latter unsymmetrical."

But also mathematically there is no valid reason why we should regard an asymmetrical relation more ultimate than a symmetrical one. A line may be generated by a transitive asymmetrical relation[44] between points, *i. e.*, a relation of the type

$$aRb \text{ implies not } bRa,$$
$$aRb \text{ and } bRc \text{ imply } aRc,$$

or a transitive symmetrical relation[45] between dyads, *i. e.*, a relation of the type

$$abKcd \text{ implies } cdKab,$$
$$abKcd \text{ and } cdKef \text{ imply } abKef.$$

Russell curiously infers from the definition of asymmetrical relations on the basis of the symmetrical that the latter are not essential.[46] Why? By applying an analogous argument to asymmetrical relations we might easily prove that these are non-essential. For example we can define[47]

$$abKcd \text{ means } (aRb \text{ and } cRd) \text{ or } (bRa \text{ and } dRc).$$

In the second case we have transitiveness and symmetry on the basis of asymmetrical relations. In geometry we should say that the two methods of generating space are equivalent. If we consider n-dimensional space generated linearly $(n > 1)$ an infinite class of transitive asymmetrical relations is required,[48] while a single transitive symmetrical relation suffices. Thus there exists in this case a practical reason for preferring transitive symmetrical relations. Russell's reduction of the latter to asymmetrical relations, by the way, is effected through a "principle of abstraction," closely allied to Peano's "definition by abstraction" which Vailati has characterized as *pragmatic*.[49] From a practical mathematical standpoint I am unable,

[44] *Cf. American Journal of Mathematics,* Vol. 31, page 378.

[45] *Cf. Amer. Jour. of Math., loc. cit.,* page 394.

[46] *Cf.* Russell, *loc. cit.,* page 235.

[47] *Cf.* Russell's view of "and" as a relation, *loc. cit.,* page 71.

[48] *Cf.* Russell, *loc. cit.,* page 395.

[49] *Cf.* G. Vailati, "Pragmatism and Mathematical Logic," *Monist,* Vol. 16, page 487. Russell uses the principle of abstraction throughout his book; see espe-

then, to verify that absolute position asymmetrical relations enjoy in Russell's book.

5. Before considering the possibility of constructing an internal theory of relations, including the asymmetrical, it will be useful to exhibit some of the inconsistencies and vagaries in the Russell-Spaulding external theory. We notice, for example, an arbitrariness and uncertainty on the part of Russell and Spaulding concerning particular relations. Russell says: "It seems best to regard *and*[50] as expressing a definite unique kind of combination, not a relation." Spaulding,[51] on the other hand, assumes that *and* does express a relation. Again Russell explicitly assumes[52] that membership of a term in a class is a relation, and this assumption leads[53] him to affirm that some relations which hold between a term and itself are not necessarily symmetrical, a statement which seems formally undesirable. Concerning identity Russell[54] says frankly:

"The question whether identity is or is not a relation and even whether there is such a concept at all[55] is not easy to answer. For it may be said identity can not be a relation since where it is truly asserted we have only one term, whereas two terms are required for a relation. . . . Identity must be admitted and the difficulty as to the two terms of a relation must be met by a sheer denial that two different terms are necessary." The conclusion that must here be drawn is that identity, as a relation, has a very dubious existence. And if identity, as a relation, is in question, the same must be said of difference, because the interdependence of identity and difference is, I think, fairly well recognized.[56] Lastly, I observe that Russell has arbitrarily assumed[57] that a sensed couple involves a relation:

"It may be doubted whether there is any such entity as the sensed couple, and yet such phrases as 'R is a relation holding from a to b' seem to show that its rejection would lead to paradoxes."

"It would seem, viewing the matter philosophically, that sense

cially *loc. cit.*, page 519; compare also page 51. See also G. Vailati, *Revue du Mois*, Vol. 3, 1907, pages 162–185.

[50] The term "and" has a far more pregnant meaning in symbolic logic than Russell recognizes (*loc. cit.*, §§ 71, 98). Consider, for instance, the definition, given above, of the relation of $abKcd$ in terms of the relation aRb.

[51] *Loc. cit.*, page 162.

[52] *Cf.* Russell, *loc. cit.*, §§ 21, 26, 30, 53, 68, 69, 76–78, 125, (*cf.* 491); see also pages 25, 167.

[53] *Cf.* Russell, *loc. cit.*, §§ 30, 57, 76, 79, 94, 95.

[54] *Loc. cit.*, pages 63–64.

[55] On page 96, *loc. cit.*, Russell says: "Self-identity is plainly a relation," but on page 163 expresses doubt about identity being a relation.

[56] *Cf.* for instance, Bradley, "Appearance and Reality," 2d edition, pages 585, 617, etc.

[57] *Loc. cit.*, pages 87–88; 99 (*cf.* page 25); 512, note; 107, note.

can only be derived from some relational proposition." Russell thus seems by no means certain that the sensed couple involves a relation.

If one assumes that identity and diversity are not relations,[58] that possession of a trait does not express relation,[59] that reference[60] to a term is non-relational, and that no relation is involved in a sensed couple or rather *functional ordered*[61] *dyad* $(x\ y)$, then it seems possible to explain asymmetrical relations on an internal basis. I will mention briefly how this might be done. As a standard form of a binary functional relation, I assume xRy, that is, "x possesses R with reference to y;" a relation between x and y arises, then, if the term x possesses a *mark* or *trait* with reference to the term y. Now I assume that xRy is always equivalent[62] to $(xy)R_1(xy)$ where (xy) is a functional ordered dyad, and that R_1 is symmetrical, *i. e.*, (xy) and its repetition may be interchanged. Therefore the preceding interpretation suggests that a binary relation may be generated by comparing an ordered dyad (xy) with its repetition; one has xRy if, and only if, (xy) possesses R_1 with reference to itself, or (xy) and its repetition possess a common mark or[63] the dyad (xy) and its repetition are "relatively equal" with reference to a mark. It should be observed that the equivalence of xRy and $(xy)R_1(xy)$ involves subtle distinctions in Russell's external theory of relation; it has as underlying problems the analysis of a reflexive relation[64] and the relation of the class of all propositions of the form xRy to the associated propositional function of two variables, $\phi(x, y)$.[65] This preceding internal theory seems consistent, but contradicts[66] several

[58] *Cf.* Bradley, *loc. cit.*, page 582.

[59] Russell, *loc. cit.*, §§ 53, 79, also §§ 425, 426.

[60] Russell, *loc. cit.*, § 214. In the above I conceive of "trait" not merely as something that may be possessed by a term, but also as something that is *relevant* to some term (*cf.* Russell, *loc. cit.*, §§ 81, 82). Reference, or rather relevance, is a preliminary that may lead to relation. On "relevance" see Schiller, *Mind*, 1912.

[61] See *Amer. Jour. of Math.*, Vol. 31, pages 370, 375.

[62] As tending to illustrate this equivalence consider "$x=y$" and "$x-y=x-y,\ x-y=0$."

[63] *Cf.* Veronese, "Grundzüge der Geometrie," pages 2–5.

[64] *Cf.* Russell, *loc. cit.*, page 86, paragraph 2.

[65] It seems possible to approach the above equivalence on the Russell basis by saying (*cf.* Russell, *loc. cit.*, page 85, paragraph 2 and § 74) $(xRy)\epsilon\phi(x, y)$, or if one is thinking of a functional ordered dyad underlying xRy one has $(x, y)\epsilon\phi(x, y)$, as equivalent to the former symbolic statement. Now $(x, y)\epsilon\phi(x, y)$ expresses a relation of a class to a class of which it is the only member, *viz.*, a class of couples (x, y) to the associated propositional function $\phi(x, y)$. One might, therefore, conveniently express $(xy)\epsilon\phi(x, y)$ in terms of a single (relational) symbol and get $(xy)R_1(xy)$. Compare also A. T. Shearman, *Mind*, 1907, page 260.

[66] I assume that subject-predicate propositions are reducible to the standard,

of Russell's controversial assumptions concerning relation and class.[67]

It seems plausible, therefore, to modify the monistic and monadistic theories of relation so as to yield, formally at least, an unobjectionable internal[68] theory. Against Russell's external theory, it might be urged that the external theory assumes[69] the definition of the general effectiveness of a relation:

"The relation affirmed between *A* and *B* in the proposition '*A* differs from *B*' is the general relation of difference and is precisely and numerically the same as the relation affirmed between *C* and *D* in '*C* differs from *D*.' And this doctrine must be held to be true of all other relations; relations do not have instances, but are strictly the same in all propositions in which they occur."

In my opinion a more correct statement would be that general concepts of relation are limit concepts to which classes of specific instances of relation sometimes tend. Aside from this, however, it may be questioned whether Russell has succeeded in entirely avoiding relations as specific instances. What I suspect to be a disguised internal relation is Russell's "measurable relation between two vectors" which he describes[70] as follows: "To say that the relation is measurable in terms of real numbers means . . . that all such relations have a (1, 1) relation to some or all real numbers." From the standpoint of the correspondence with real numbers it seems altogether likely that we are concerned here with specific relational instances in which the terms related are peculiarly involved. A precisely analogous relational problem occurs elsewhere in the abstract mathematical science of Grassmann, the *Ausdehnungslehre* which has geometry and mechanics for particular applications. On the introduction of the number system into his discipline Grassmann states[71] explicitly: "The numerical magnitude as developed in our science does not appear as discrete number, *i. e.*, not as a set of units, but . . . as a quotient of continuous magnitudes and therefore does not at all presuppose the discrete conception." This conception of number, regarded in a certain way as a foundation for a general theory, may have its limitations; nevertheless, its use is, I think,

equivalent forms, "*a* possesses *M*," "*a* belongs to *C*," "*O* affects *a*," where *M*, *C*, *O* may lead to a relation, class, operation, respectively. See Russell, *loc. cit.*, §§ 57, 79.

[67] See, for instance, "Principles of Mathematics," page 167, paragraph 1.

[68] I do not wish to imply here that I uphold a purely internal theory of relation. On the contrary neither an internal theory nor an external theory, in itself, appears to me adequate.

[69] Russell, *loc. cit.*, § 55, page 51.

[70] *Loc. cit.*, page 433.

[71] *Cf. Gesammelte Werke*, Vol. I., page 138.

amply justified in Grassmann's Extensive Algebra and constitutes a serious difficulty in the purely external theory of relation.

6. Reverting for a moment to the falsification of analysis, I may be permitted to indicate how conflict, which is the source of embarrassment to analytical realists, is employed to advantage in the pragmatic position. To have recognized the fundamental part of conflict in the process of knowledge is, I believe, one of the great merits of pragmatism.[72] This philosophy, at least in regard to conflict, seems more nearly in accord with the facts of mathematics as an incomplete science than analytical realism. Let me review briefly a few important instances of conflict in the history of mathematics and the developments to which they have given rise. It is proper here to quote Hilbert:[73]

"In modern mathematics the question of the impossibility of solution of certain problems plays an important rôle and the attempts made to answer such questions have often been the occasion of discovering new and fruitful fields for research. We recall . . . the demonstration by Abel of the impossibility of solving an equation of the fifth degree by means of radicals, as also the discovery of the impossibility of demonstrating the axiom of parallels, and finally the theorems of Hermite and Lindemann concerning the impossibility of constructing by algebraic means the numbers e and π."

Again, Hamilton endeavored to construct an algebra of three units, $a + ib + jc$, which should obey the same laws of operations as the ordinary complex number, $a + ib$; and out of the conflicts that arose between these algebras, as he describes in detail in the preface to his "Lectures on Quaternions," he was led to construct a new complex number of four units, the quaternion. If I may give another example, the problem of the continuity of the straight line is that presented by the conflict of an intuitive straight line, say L, with the class R of rational numbers; namely, on the straight line L there is an arbitrary number of points which corresponds to no rational number, while to every rational number there corresponds a point. Thus as Dedekind says in his celebrated memoir,[74] a comparison of the intuitive straight line L with the rational numbers R shows that the

[72] *Cf.* G. H. Mead, *Philosophical Review*, Vol. 9; A. W. Moore, "Pragmatism and its Critics," page 125. See also Stosch, *Vierteljahrschrift für wiss. Phil.*, Vol. 29, page 97, note 3.

[73] "Foundations of Geometry," page 131. Compare O. Perron, "Ueber Wahrheit und Irrthum in der Mathematik," *Jahresber. d. Deutsch. Math. Ver.*, Vol. 20 (1911), page 196; H. Liebmann, "Nothwendigkeit und Freiheit in der Mathematik," same journal, Vol. 14 (1905), page 230.

[74] "Stetigkeit und irrationale Zahlen," pages 7–11.

latter presents gaps, but the former does not. Dedekind's solution of the conflict was his formulation[75] of the principle:

"If all the points of a line are separated into two classes such that every point of the first class lies to the left of every point of the second class, then there exists one and only one point which produces this separation." More generally, I might refer to the conflict between analysis and geometry in the development of mathematics and that great movement initiated by Lagrange known as the arithmetization[76] of mathematics in which rival theories due to Cauchy and Weierstrass and Méray are prominent. But the examples given will, I think, sufficiently indicate an important aspect of mathematics as heuristic.[77]

IV

I come now to criticisms of a more properly mathematical nature. One of Professor Spaulding's colleagues, Professor W. B. Pitkin, in the "New Realism" (p. 378), speaks of the objections which mathematics has brought against realism in the past, and implies that these have been cleared away in previous essays in the volume, presumably Spaulding's essay. This statement, alas! can not be verified. Spaulding has committed many mathematical errors and it is proper to state that these seem due to an unfamiliarity with mathematical conceptions, rather than to the peculiar philosophic position he upholds; his mathematical remarks obscure rather than elucidate his fundamental theses. On this account I enter upon a mathematical criticism of Spaulding's essay rather unwillingly, and a few indications of the most noticeable mistakes must suffice.

Readers of Spaulding's essay will quite agree with the author when he says,[78] "It is important . . . to present clearly and with precision that which analysis shows the continuum to be." But this is what Spaulding does not do. (1) Consider his statement (p. 178), "That there are irrationals is discovered in the realization that there

[75] Obviously, Dedekind's statement lacks rigor; *cf.* Russell, *loc. cit.*, § 266.

[76] *Cf.* G. Bohlmann, *Jahresbericht der Deutschen Mathematiker Vereinigung*, 1901, page 95.

[77] In the preceding section it might have been desirable to dwell on the nature of conflict in general. Research mathematicians will probably have no difficulty in recognizing in their own experiences what is meant by the *conflict* of mathematical terms. The subtle character which such conflict often possesses may be illustrated by an example. The statements, "$1+2=3$" and "$9+16=25$" are not in conflict in reference to addition of integers, but they are in conflict in that $9+16=25$ may be expressed $3^2+4^2=5^2$, while $1+2=3$ does not admit an analogous expression in terms of squares. More broadly, there is a conflict between "intuitive" and "formal" mathematics.

[78] *Loc. cit.*, page 78.

is some value for x whereby, for example $x^2=2$. The position[79] that underlies this statement is incorrect. What probably suggested this remark to Spaulding is the possibility of making the (intuitive) construction which he has described (p. 184, paragraph 3).[80] (2) The definition of limit of a sequence Spaulding misquotes[81] from Pierpont's book.[82] I remark that Pierpont's theory of real numbers has been discussed in a review by G. A. Bliss.[83] (3) The error is committed by Spaulding (p. 179) of juxtaposing the *derivative of an assemblage* and the *derivative of a function;* further comment on this seems unnecessary.

Spaulding's analysis of space is not more satisfactory than his arithmetical analysis. (1) Spaulding seems to have been misled (p. 184, paragraph 1,) by Hilbert's use of the word "continuity" in connection with the Archimedean property of a line.[84] (2) The author's inability to comprehend mathematical continuity is clearly shown, as is evident elsewhere, by his remark (p. 185) that a "series" is continuous if it is perfect. The latter remark is contradicted by *nowhere dense perfect assemblages.*[85] (3) Spaulding makes a very feeble attempt (p. 188) to explain the relation of the extension of a line to its continuity. What we are concerned with here is the dependence of the Archimedean axiom on the axiom of Dedekind continuity in the foundations of geometry; this has been recently discussed by O. Hölder.[86] In view of such instances as the preceding, Spaulding is not justified in saying[87] that his analysis of space "states with clearness and precision what space is, what its continuity is, what terms and relations are involved."

I shall not consider further Spaulding's errors. Enough has been

[79] Russell has a remark quite as misleading as the one quoted above from Spaulding; see Jourdain, *Math. Gazette,* Vol. IV. (1908), page 204, note.

[80] Compare also Dedekind, *loc. cit.*

[81] *Loc. cit.,* page 178.

[82] "The Theory of Functions of Real Variables," Vol. I., page 25, § 42; page 61, § 97.

[83] *The Bulletin of the American Mathematical Society,* Vol. 13 (1906–1907), pages 121–122. See especially Bliss, *loc. cit.,* page 121, note; compare H. Weber, *Jahresber. d. Deutsch. Math. Ver.,* Vol. 15 (1906), page 173. Instructive references to the theory of real numbers are, A. Pringsheim, preceding journal, Vol. 6, page 73; O. Perron, same journal, Vol. 16 (1907), page 142, and Jourdain, *Math. Gazette,* Vol. IV. (1908), page 201.

[84] "Foundations of Geometry," page 24.

[85] See Schoenflies, *Jahresbericht der Deutschen Mathematiker-Vereinigung,* Vol. 8 (1900), pages 101–102; *cf.* Russell, *loc. cit.,* page 440, § 417; page 288, § 272.

[86] *Leipziger Berichte,* Vol. 63 (1911), pages 108–109. *Cf.* K. Th. Vahlen, *Jahresber. d. Deutsch. Math. Ver.,* Vol. 16 (1907), page 409.

[87] *Loc. cit.,* page 185.

said to establish that he not only is unable to analyze successfully those mathematical contents with which he has dealt, but he does not even possess familiarity with the mathematical conceptions he mentions. In this respect Spaulding differs from Russell, who, I think, has been reasonably accurate in dealing with mathematical conceptions, and who has been aptly characterized by James[88] as an "athletic ratiocinator." But so far as I have been able to ascertain, neither Spaulding nor Russell has had experience in mathematical research, and discovery, as Peirce[89] has pointed out, is a part of mathematics.

V

In conclusion, I should like to lay stress on the desirability of a more intimate relation between philosophers and mathematicians. "The mathematician's interests," says Royce,[90] "are not the philosopher's. But neither of the two has a monopoly of the abstractions and in the end each of them—and certainly the philosopher—can learn from the other. The metaphysic of the future will take fresh account of mathematical research." The numerous misinterpretations of mathematics occurring constantly in philosophical literature, probably not excepting the work of Bergson,[91] show that philosophers can not pronounce judgment on mathematical contents without acquainting themselves with mathematics in a way that probably requires actual mathematical experience. Conversely, mathematicians should endeavor to enter into the spirit of philosophical disciplines and recognize that the study of philosophy can be made indirectly the means of further mathematical development.[92] Mathematical masters have sometimes acknowledged explicitly this advantage of philosophic study. It is said of Kronecker,[93] for instance, that he thought more philosophically than mathematically and considered it profitable to go beyond his special mathematical field, to aim at general ideas, and then to return to his more restricted activity.

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88 "The Meaning of Truth," page 276.

89 *Cf.* J. B. Shaw, *Bull. Am. Math. Soc.*, Vol. 18, page 381.

90 "The World and the Individual," Vol. I., page 527; compare Vol. II., page x.

91 *Cf.* E. Borel, *Rev. de Met. et de Morale*, Vol. 16, 1908, pages 244–245.

92 *Cf.* M. Winter, *Rev. de Met. et de Morale*, Vol. 16, 1908, page 920.

93 *Cf.* Netto, Mathematical Congress Papers, Chicago, 1893, page 243; see also page 246.

VALUE IN ITS RELATION TO MEANING AND PURPOSE

IN philosophical discussions it is common to find the terms meaning, purpose, and value bracketed together, or used interchangeably. Thus, in introductory text-books of philosophy such statements may be found as that science has for its problem the description and explanation of facts, philosophy the interpretation of facts in terms of meaning, of purpose, or of value; or that the scientist is interested in facts as such, whereas the philosopher inquires further into what these facts mean, what they are for, and what function they fulfil in the universe. But though considerable thought has been and is being given to the definition of each of these concepts, the investigation of their mutual relations has been neglected, and it is to this phase of the matter that I wish here to call attention.

In the first place, the three concepts have in common the factor of external reference. However much independent reality we may attribute to any object, to assert that such an object has a meaning, purpose, or value is to acknowledge its lack of complete independence. All three concepts, then, are determined by some relation, the subsistence of which is essential to the adequate definition of the object concerned; and if this is so, we must look to the other term of the relation for the principle of differentiation between them.

Now the meaning of an object is always determined by its relation to some other object. When any object A has a relation to another object B, such that the subsistence of that relation is essential to the adequate definition of A, B is determined as the meaning of A, A as the "sign" of B. Thus, a frown is not merely a contortion of the face, but a sign of anger, of deeply concentrated attention, of anxiety or uncertainty, etc.: that is to say, the presence of the frown is an indication of the existence of some definite state of mind in the individual which "expresses itself" through the frown, and without which the latter would be meaningless. So, the sound of the dinner-bell ordinarily means that dinner is ready, and without a dinner ready for consumption the ringing of the bell is meaningless. So, again, the motif of Beethoven's Fifth Symphony is said by some to mean "Fate knocking at the door," by others to represent (*i. e.*, to mean) the crying of a baby, and by others still is regarded as no more than a theme to be developed, *i. e.*, as having its meaning only in relation to that which follows in the composition.

Meaning has reference to an accomplished fact, purpose to the accomplishment of some fact: the meaning of an object is determined by its relation to some other object as existent, purpose by its relation to some change in or modification of some object. Thus the purpose of the nomination of a fusion ticket in New York City last

summer was to defeat Tammany, the meaning of the election of Mr. Mitchel was the accomplished defeat of Tammany. Hence the definition: when any object A has a relation to a modification of some object B (as B'), such that the subsistence of that relation is essential to the adequate definition of A, then the change from B to B' is determined as the purpose of A. If the meaning of a frown is anger, its purpose is to convey the information that the frowner is angry, and so far to produce a change in the mental state of the observer: the angry man may suppress the frown, but if he frowns his facial expression fails to fulfil its purpose if no modification results therefrom in the mind of the observer. So the purpose of the dinner-bell is to produce, let us say, a movement of the guests from the conservatory to the dining-room; the purpose of the Fifth Symphony motif to arouse the notion of Fate or the image of a crying baby in the minds of the hearers, or at least to prepare them for the musical mood of the composition.

Value, finally, is determined by the relation of the valuable object to a subject, and is defined from the standpoint of the subject rather than, as in the former cases, from that of the object said to "possess" value. When any object A has a relation to a subject S such that the subsistence of that relation is essential to the adequate definition of S, A is said to be valuable, and the relation itself to constitute the value of A. Thus the frown, the dinner-bell, the Beethoven motif, the New York election, have value because they in one way or another fulfil the interests, satisfy the needs, or complete the reality of some subject. The frown is valuable because it enables the angry man to express his anger, for without this or some other outward sign the needs of the subject would fail to find satisfaction: the dinner-bell is valuable because it enables the server to summon the diners with less effort than any other method; the Beethoven motif is valuable because it expresses the composer's and arouses the hearers' esthetic sentiment; the New York election, finally, is valuable because it expresses in the most practical and efficacious manner the dissatisfaction of the people with political conditions.

These definitions, it is hoped, bring out clearly the close interrelations between our three concepts, and justify the use of them as alternative predicates of objects whose reality extends beyond their isolated existence. Purpose is hardly distinguishable from meaning when predicated of an action, and value has been defined by Urban as "affective-volitional meaning." Whether any fact that possesses meaning must also possess purpose and value may be questioned, but whatever has purpose or value has thereby meaning also. Meaning is defined in terms of relation to some object—but if we think of the denotation of "object" as including also changes and conscious

selves, purpose and value become species of meaning. In other words, if we conceive meaning as genus, and regard it as divisible into the two species, logical (or cognitive) and affective-conative meanings, purpose and value become subspecies under the second head—purpose essentially conative meaning, value, primarily, perhaps (but not solely), affective.

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REVIEWS AND ABSTRACTS OF LITERATURE

La Synthèse Mentale. GEORGES DWELSHAUVERS. Bibliothèque de Philosophie Contemporaine. Paris: Librairies Felix Alcan et Guillaumin. 1908. Pp. 276.

The book, as its title indicates, is a study of the mental life. It is an exposition of the nature of consciousness which Dwelshauvers characterizes in terms of a synthetic activity. Its object is to explore the content of this synthetic activity, to define that act which constitutes "the unity of mentality." It is an attack on the materialistic and sensationalistic theories that have reigned in psychology for so many years. Dwelshauvers does not pretend to have originated the problem of the mental synthesis; it is a continuation, but through a different method, of the line of work which in France has preoccupied thinkers like Maine de Biran, Ravaisson, Lachelier, Paulhan, Ribot, Pierre Janet, and many others. The fundamental idea of Höffding's "sketch of a psychology founded on experience" is, as Pierre Janet remarks in his preface to the French edition, that "consciousness is essentially an effort toward unity, a synthetic force," and that even elementary sensations,—and not only the higher types of judgment and general ideas,—are synthetic in their nature. Dwelshauvers's work, though of a different scope, is based on those fundamental assumptions.

Dwelshauvers is brought to the notion of synthesis not by metaphysical reasons, but, as he claims, through the simultaneous use of diverse methods of psychology applied to the mental fact which he attempts to define in its concrete reality, and not in abstraction. What Dwelshauvers calls the unity of the life of the mind has nothing in common with the Cartesian theory of a soul unable to have more than one idea at a time on account of its simplicity, nor with the unity of the soul such as Herbart has admitted. The unity which is presented by the mental synthesis is, according to Dwelshauvers, neither an abstract entity nor the property of a substance, but the realization more or less perfect of a complex equilibrium maintained among diverse tendencies which constitute the conscious individuality. "The synthesis of the life of the mind," to quote our author, "is neither a combination of elements nor simply an expression of logical relations, but it is an act which explains the natural way by which all of us, whether ignorants or subtle analysts, posit our ego

at the same time as we posit as equally real, *i. e.*, as subjects, other egos, or the non ego." "This act is intuitive, it is prelogic, it answers to an interior vision, to a real affirmation of faith, or more simply, of sympathy."

The book is divided into four chapters which deal respectively with (I.) the mental and cerebral activity, (II.) the unconscious, (III.) the mental life and the laws which govern it, (IV.) personality and freedom.

In the first chapter the author examines the nervous system in its relation to thought and shows that the physiology of the brain does not explain the act of thought. The nervous cells do not produce representation; representation is the work of an act of the mind. It is an interior act issued from this pure and non-spatial potentiality which mind is. The act of the mind surpasses in its richness and variety the cerebral activities which accompany it. The brain is to consciousness what the piano is to the musical artist; the instrument has a restricted number of keys, consequently the number of its movements is very limited. However, what the artist will play will vary infinitely. Viewed from its inner side, the life of the mind appears as an élan, an interior movement; studied in its relation with the organism which it animates, the life of the mind manifests itself as anticipation. It anticipates the nervous system, it establishes means of communication and does not make use of the means already in existence. It is essentially effort, *inextensivity,* dynamism. A mind which would be merely a synthetic consciousness of the organism would not create anything new, would not adjust itself to unexpected adaptations, in a word it would not live. Dwelshauvers, like Bergson, rejects the psychophysical parallelism and, like him, criticizes those psychological systems which speak of cerebral images, aggregates of sensation in the brain and of psychophysiological localizations. He sums up the first chapter by emphasizing the fact that we lose sight of the real problem by attributing to the brain a function which it does not possess, namely, "the formation of images, the activity of thought or ideation."

The second chapter deals with the unconscious in the mental life and its relation to conscious thought "whose continuity is assured by the movement and the depth of the unconscious life which never stops and never tires" (p. 115). The author makes an interesting study of the different forms of the unconscious, from the unconscious in the act of thought to the unconscious in the affective life. He groups them into two classes, the ultra-psychic unconsciousness and the psychic unconsciousness. The first comprises the two limits of our mental life, the rational unconsciousness which manifests itself in the act of the mind and the irrational unconsciousness which connects itself to the organism. The psychic unconsciousness manifests itself in memory and in automatism. The conclusion of this study, which is carried over to the third chapter, is that the mental life is not a series of states, "but is formed of an indeterminable number of psychic currents of different force and quality upon which there falls an ever changing illumination with an infinitely varied play of light and shade. These currents sometimes diverge, sometimes go parallel, and sometimes rejoin in order to divide again."

So far the author has endeavored to give us a conception of the mental life. He now tries, in the rest of the third chapter, to consider the laws that govern the mental life. This brings him to a consideration of the categories of quality, quantity, duration, causality, and finality, those objective categories which we are tempted to carry from the objective world into the interpretation of the mental life. He shows what a profound transformation the specificity of the spiritual life imposes on those categories. The spatial categories as well as objective causality can not apply to the mental life. Mental life differs from phenomena in this essential point that all prevision with regard to it is impossible. Causality for the psychologist does not connect phenomena and does not form an indefinite series, but unites concrete facts, ideas, sentiments, volitions to a personality. Nor is it legitimate to reduce the psychic finality to the finality applicable to the external world. They have some common characteristics, but consciousness and the will bestow upon psychic finality an altogether different value. "The life of the mind," says Dwelshauvers, "escapes the logical determination in which our reason groups phenomena. It can not be understood without a theory of freedom." This theory of freedom is the subject of the fourth chapter.

The psychological problem of liberty has no solution, according to our author, as soon as we ask with regard to a given act whether that act is free or not. Freedom has sense in psychology only for a series of acts, for the ensemble of an activity, and not for such and such an act in particular. The more a voluntary act is part of a more unified movement the more free it is. It is the force of cohesion and resistance of the ego which gives the measure of liberty. "One who prefers the unity of his interior development to the solicitations of the environment tends to act freely." Freedom is thus explained by personality and synthesis. It is that unity which is the equilibrium, the harmony, of the different tendencies that make up the conscious individuality which explains freedom. That unity, as we have already seen, is the mind, the spiritual principle par excellence, which reveals itself in the pure act, in intuition, and in reflection.

Here the author gives a historic sketch of the notion of mental synthesis held by those philosophers whose writings help us to understand better the meaning of the spiritual life. He takes up Leibniz, Kant, Wundt, Höffding, Pierre Janet, and Bergson. But it is rather surprising that the author has omitted from this list Fichte, whose doctrines, if translated in contemporary terms, have, it seems to me, something in common with those of Dwelshauvers.

The book contains, in addition to those four chapters, an appendix which gives us a critical review of the various methods used in psychology. The method with which Dwelshauvers identifies himself is the reflective method, so named and defined by Jules Lagneau. This method, as we can see from the plans followed in our present book and as Dwelshauvers describes it himself, has for aim the explanation of the logical unity which consciousness presents, the determination of its essential characteristic and the search for the laws which would enable us to under-

stand them. Dwelshauvers claims that this reflective method is superior to the other ones because it has not the shortcomings of the introspective and the psychophysical methods; that it is at the same time rational and living, rational because it brings all psychological manifestations to a unity, and living because it seeks the condition of the real,—it makes no abstraction. Dwelshauvers's method differs from the Bergsonian intuition. For the Bergsonian intuition aims to reach the bottom of the spiritual life under the stratified layers of the logic and the social, while the reflective analysis applies itself more specially to thought as ideation and reflection. It is by this method that, according to the author, we arrive at a most correct notion of the mutual implications of ideas in the concrete totality which constitutes all act of thought, of cognition, or of reflected volition.

Such is but a brief outline of the fundamental ideas underlying the book, the central thought of which is the revindication of the specific and autonomous character of our mental life. The conclusions which Dwelshauvers reaches in his work do not differ much from those of Bergson. But he comes to them through his own method, through his own original and personal way of thinking, based on deep reflections, on scientific researches, and on accumulation of facts which have been furnished to him by the experimental investigations of leading scientists. Whether or not we admit the author's theory of consciousness, we can not fail to find his book most interesting and suggestive. It is at once a contribution to and excellent representative of contemporary psychology.

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The Philosophical Works of Descartes. Rendered into English by ELIZABETH S. HALDANE and G. R. T. ROSS. Two volumes. Cambridge University Press. 1911. Vol. I. Pp. vi + 452; Vol. II. Pp. viii + 380.

This edition makes accessible to English readers much which has been overlooked in other English renderings of Descartes. Thus far English translations have been limited almost exclusively to the "Discourse," the "Meditations," and selections of the "Principles." These works are, however, not enough to give us a comprehensible view of Descartes. The way in which he expounded his theories makes a more extensive acquaintance with his works necessary for a thorough understanding of his philosophy. Descartes never expressed freely and openly what he believed to be the truth. He gave us his ideas only in disguise; his progressive theories are veiled in conservative covers. The "Discourse," the "Meditations," and the "Principles" only arouse our suspicion of a double policy on the part of Descartes; a systematic study of his scientific treatises and his other works confirms this suspicion, and, what is more important, enables us to free the kernel of his philosophy from the husk. The present translators, E. S. Haldane and G. R. T. Ross, have thus rendered an invaluable service to English readers by including some of the scientific treatises and the polemics in this edition.

Of the scientific treatises, Volume I. contains the "Rules for the Direc-

tion of the Mind" and "The Passions of the Soul." The "Rules" give us the original sketch of Descartes's method and its application to mathematics. In this treatise the explanation of Descartes's view of extension is worthy of notice. He refutes the independent existence of extension and explicitly states that while "body possesses extension," "extension is not body" (pp. 57 sq.).

"The Passions of the Soul" is a mechanistic interpretation of all vital phenomena. This was the first attempt to apply the mechanical principle to both mental and physiological processes. Present-day psychology and physiology testify to the significance of this attempt. English readers have been thus far deprived of this very interesting treatise, as the first translation of 1650 is practically out of print at present.

Of other works that have been given here their first English rendering, Volume I. contains "The Search after Truth" and "Notes directed against a Certain Programme."

"The Search after Truth" is an unfinished dialogue which exemplifies the search for truth according to the Cartesian method by the "natural light" alone.

The "Notes" is a refutation of ideas expressed in a pamphlet on the nature of the mind, edited anonymously in the form of a manifesto or poster by Regius. These ideas conflicted with orthodoxy, and as Regius was generally known as an ardent follower of Descartes, the latter publishes his opposition to a very much feared denunciation. In these "Notes," translated without the preface and the verses that accompanied the original, Descartes's incidental explanation of the innate ideas is very elucidating, and makes all objections against them in the history of philosophy appear to us vain (p. 442).

In addition to the above-mentioned works, Volume I. contains the "Discourse," the "Meditations," and the "Principles."

The "Principles" are here more fully translated than in the previous English editions. The headings of all passages not translated are given, so that the contents can be inferred.

The second volume of this edition comprises the "Objections and Replies," "Arguments Demonstrating the Existence of God," a "Letter from Descartes to Clerselier," and a "Letter to Dinet."

In the Introduction to this volume the translators explain the origin of the "Objections and Replies." Before publishing the "Meditations," Descartes circulated it in manuscript among various theologians and philosophers. Their criticisms and Descartes's replies were later published, together with the "Meditations."

The "Objections and Replies" consists of seven sets of objections and replies; objections by (I.) the theologian Caterus, (II.) a group of theologians and philosophers, (III.) Hobbes, (IV.) Arnauld, (V.) Gassendi, (VI.) another group of theologians and philosophers, and (VII.) Bourdin.

The "Objections" are criticisms, on one hand, from the theological, and, on the other, from the empirical and scientific, points of view of Descartes's following doctrines: The proofs of God's existence; efficient causality and "*causi sui*"; continual creation; liberty of indifference in

man and God; the method of doubt; the principle of definition as criterion; the distinction between soul and body; the nature of the "thinking thing," and the spirituality of the soul, and the identification of substance and accident.

These criticisms contain things of considerable interest. In the "Objection" by Caterus we find a striking parallel drawn between Descartes's ontological argument and that of St. Tomas (pp. 3 sq.). In the "Objection" by Arnauld, we have the identification of Descartes's "*je pense, donc je suis*" with St. Augustine's "*si fallor sum*" (p. 82), and the discussion of Descartes's likeness to St. Augustine in the doctrine that the soul is more clearly perceived than the body. In the objections of Arnauld—"as a theologian," perhaps the most interesting points are his explanations of the danger of Descartes's rule of evidence for the teachings of theology, and of the incompatibility of Descartes's conception of matter with the theory of the Eucharist (pp. 93 sq.).

Gassendi's "Objections" gives us an insight into his materialism and the reaction of an empiricist to Descartes's assertion that the mind is more clearly perceived than the body, and that its essence is mind, and to Descartes's similar speculative doctrines. Hobbes's "Objections" is gratifying as an exposition of the relation of this materialistic thinker to Descartes's spiritualistic philosophy. In these objections (pp. 61 sq.) the interpretation of Descartes's conception of the "thinking thing" as favorable to the soul's materiality attracts our attention as we meet with the same objection in Gassendi's criticism.

In Descartes's "Replies" to the "Objections" we have elaborate discussions of his doctrines criticized by the opponents. In these "Replies" comes up Descartes's attempt to reconcile his theory of matter with the teaching of the Eucharist (pp. 116 sq.), which is later more elaborated in a letter to Mesland.—Descartes's distinction between real and formal existence in the case of everything except God, is brought out here more definitely than in the "Meditations" (especially p. 20).

The "Objections and Replies" are not important as an elucidation of the difficulties with which we meet in Descartes's speculations; for in his replies to the opponents he makes various concessions. This work is interesting rather as an illustration of the attitude of Descartes's theological and philosophical contemporaries towards his philosophy, and of his efforts to justify the difficulties of his speculations leading to the accusation of heterodoxy on one hand, and lack of empiricism on the other.

The "Arguments for God's existence" is an exposition in geometrical form of the same proofs as occur in the "Discourse," the "Meditations," and the "Principles."

In the "Letter to Father Dinet" Descartes complains of accusations that his ideas are opposed to ancient philosophy and clash with theology, and expresses his eagerness for the approval of his ideas by the Jesuit society, of which approval he despairs on account of Bourdin's attack.

The "Letter to Clerselier" consists of Descartes's replies to objections made by Gassendi to previous replies.

The rendering is done very carefully; expressions that can not be

translated precisely are quoted in the footnotes; literal exactness is, however, often sacrificed for the sake of English style. The variations in the text of the different editions of the original works are indicated by means of brackets.

The translators introduce each work by valuable notes as to place, date, and circumstances of publication, character of the work and its history; and indicate each time what edition of the original work they followed in the translation.

In the preface to the first volume the translators promise an English rendering of Descartes's correspondence in the near future. I should say his letters are quite indispensable for an unmistakable conception of Descartes. I should also add that a translation of the treatise "Le Monde" would be very desirable in order to have Descartes in his full greatness.

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Immanuel Kants Werke. Gesamtausgabe in zehn Bänden und zwei Erläuterungs-bänden. In Gemeinschaft mit HERMANN COHEN, ARTHUR BUCHENAU, OTTO BUEK, ALBERT GÖRLAND, B. KELLERMANN, herausgegeben von ERNST CASSIRER. Verlag von Bruno Cassirer, Berlin. Bd. I. Vorkritische Schriften, herausgegeben von ARTHUR BUCHENAU. Pp. 541.

It seems remarkable that an age which, according to his own avowal, was entirely dominated by Kant, should not have given us a satisfactory edition of Kant's works, one which every one could afford to buy. For the editions of Rosenkranz and Hartenstein, apart from the fact that they are inadequate, have long since been out of print; the Kirchmann edition, although recently improved, is very uneven; the new Akademie edition is exorbitant in price, and Reclam offers only a few of Kant's writings. Under these circumstances, we note with pleasure that a number of the most prominent German Kant scholars of to-day have united to compile a new edition of Kant, which has the advantage of being not only complete and good, but also of being sufficiently cheap to be within the reach of every one. The plan calls for ten volumes and two supplementary volumes, of which the first will be a presentation of Kant's life and teachings, by Ernst Cassirer; the second, by Hermann Cohen, will concern itself with Kant's influence upon science and culture. Naturally, the treatment of the text is based upon the original Kant edition, particularly upon the last edition which Kant himself helped to compile. The manuscripts have been compared as far as they were accessible. Corrections have been made only in those cases where the discrepancies and the misprints were too evident—a note always being made to the change, however. The new style of orthography, and alas!, also of punctuation, is employed. The language, however, is altered only when rendered necessary on account of a possible misunderstanding of the sense. The outer equipment of this new edition is, in spite of its moderate cost, so rich as to make it a pleasing piece of artistic workmanship in the library of "booklovers."

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UNIVERSITY OF KÖNIGSBERG.

JOURNALS AND NEW BOOKS

THE AMERICAN JOURNAL OF PSYCHOLOGY. October, 1913. *The Measurement of Attention* (pp. 465–507): KARL M. DALLENBACH. – An experiment to measure the attention to auditory stimuli in terms of clearness values. The introspective attention values were closely correlated with objective measurements. Two types of individuals in regard to attention were found, "dual-division" and multi-level. *A Bibliography of Rhythm* (pp. 508–519): CHRISTIAN A. RUCKMICH. *Clinical Notes on the Emotions and Their Relation to the Mind* (pp. 520–524): GEORGE HENRY TAYLOR. – Emotions have their origin in sex, while opposed to emotion is reason. *A Rapid and Accurate Method of Scoring Nonsense Syllables and Words* (pp. 525–531): DARWIN O. LYON. *Characteristic Differences between Recall and Recognition* (pp. 532–544): H. L. HOLLINGWORTH. – An experimental study bringing out the importance of recognition as opposed to recall; data on the effect of the presentation, primary, recency, etc., have been presented. *A Note on the Relation and Esthetic Value of the Perceptive Types in Color Appreciation* (pp. 545–554): E. J. G. BRADFORD. – Four types of apperception were found approaching esthetic value in the following order, (1) sensational-associative, (2) physiological, (3) emotional-associative, (4) character. *The Comparative Value of Various Conceptions of Nervous Functions Based on Mechanical Analogies* (pp. 555–563): MAX MEYER. – The writer presents several simple mechanical analogies that can easily be translated into psycho-physiological terms. *An Introspective Analysis of the Association-Reaction Consciousness* (pp. 564–569): EMILY T. BURR AND L. R. GEISSLER. – There exists a close parallel between the consciousness of concealing complex and association reactions under negative instruction. *"The Feeling of Being Stared At"—Experimental* (pp. 570–575): J. E. COOVER, PH.D. – The groundless "feeling of being stared at" is experienced by one half of the student group of the university, due to a nervousness and anxiety concerning one's looks and the attributing of objective validity to subjective impressions. *Projection of the Negative After Image in the Field of the Closed Lids* (pp. 576–578): FRANK ANGELL. – A discussion of the relation of the results published by the above author to those of Mayerhausen (*Graefe's Archiv,* 1885). *Prof. Martin on the Perky Experiments* (p. 579): E. B. TITCHENER. *Minor Studies from the Psychological Laboratory of Vassar College. The Effect of the Interval Between Repetitions on the Speed of Learning a Series of Movements* (pp. 580–583): MILDRED BROWNING, DOROTHY E. BROWN, AND M. F. WASHBURN. – There are indications that the law of distributed repetitions holds in habit formation especially for complex processes. *A Suggested Coefficient of Affective Sensitiveness* (pp. 583–585): HELEN CLARK, NEIDA QUACKENBUSH, AND M. F. WASHBURN. – There seems to be little correlation between affective sensitiveness and the corresponding kinds of tests in sense impressions. *A Bibliography of the Scientific Writings of Wilhelm Wundt* (p. 586): E. B. TITCHENER AND W. S. FOSTER. *Book Reviews* (pp. 587–595): Alexander Philip, *The Dynamic Foundation of Knowl-*

edge: B. H. Bode. S. S. Colvin, *The Learning Process:* W. S. Foster. Eben Fiske, *An Elementary Study of the Brain, Based on the Dissection of the Brain of the Sheep:* W. S. Foster. R. Munroe, *Paleolithic Man and Terramara Settlements in Europe.* A. Wood, *The Physical Basis of Music.* Eugenio Rignano, *Qu'est-ce le raisonnement:* Theodate L. Smith. P. A. Talbot, *In the Shadow of the Bush. Book Notes* (pp. 596–599): C. S. Bluemel, *Stammering and Cognate Defects of Speech.* F. B. Jevons, *Personality.* Edward L. Thorndike, *Educational Psychology. Volume 1. The Original Nature of Man.* H. v. Buttel-Reepen, *Man and His Forerunners.* Leroy Walter Sackett, *The Canada Porcupine: a Study of the Learning Process.* George Rouma, *Le Langage Graphique de l'Enfant.* Hrs. v. Karl Marbe, *Fortschritte der Psychologie und ihrer Anwendungen.* Miss Etta De Camp, *Return of Frank R. Stockton.* J. Dejerine and E. Gauckler, *The Psychoneuroses and Their Treatment by Psychotherapy.* Isabel Hornibrook, *A Scout of To-day. Psychology and Philosophy* (p. 600). *Index* (pp. 601–605).

Grabmann, Martin. Der Gegenwartswert der Geschichtlichen Erforschumg der Mittelalterlichen Philosophie. Wien: B. Herder. 1913. Pp. vi + 94. $.45.

Jacoby, Günther. Die "Neue Wirklichkeitslehre" in der Amerikanischen Philosophie. Berlin. Pp. 22.

Walter, Johnston Estep. Nature and Cognition of Space and Time. West Newton, Pa. Johnston and Penney. 1914. $1.35.

NOTES AND NEWS

Letter from Dr. Schiller

To the Editors of the Journal of Philosophy, Psychology, and Scientific Methods:

As Professor J. P. Turner has done me the honor to quote, in his review of Mr. D. L. Murray's "Pragmatism,"[1] from my preface to that work a fragment which stops short of the point of my argument, *viz.*, that given a certain sort and degree of intelligence there is nothing like an intellectualistic education to develop "a perception of the *intellectual necessity* of Pragmatism" and to opine that it deserves a wide circulation, may I point out that his quotation is not quite accurate and may possibly mislead? I did not ascribe to the British nation a contempt for "*the* pure intellect", but for "pure intellect", and pointed to the practical success of administrators selected by an examination so gloriously irrelevant to their "*future* duties" as that for the British, and, until recently, that for the Chinese, Civil Service. But though I noted the paradox, I did not dispute the success, and it should, in my opinion, be a serious concern of political philosophers to account for the success of the

[1] This Journal, Vol. XI., page 24.

mandarinate in governing China for three thousand years and in attaining a stability so much in excess of any that usually befalls the institutions of man.

Very truly yours,
F. C. S. SCHILLER.

OXFORD UNIVERSITY,
February 3, 1914.

AT a meeting of the British Academy on January 28, Professor S. Alexander read a paper on "The Basis of Realism." By insisting on the equal claim of objects with the mind to be considered real, realism seems at first sight to depress the mind, and make it less real. But this misapprehension rests upon the mistake of confusing reality with perfection. Mind is not more real than things, but more perfect, *i. e.*, more developed. In view of Mr. Bosanquet's recent criticism of realism (Adamson Lecture, 1913), and to show that by depriving mind of its pretensions realism actually establishes the perfection of mind, it seemed well to restate the position. The starting-point is the analysis of an act of cognition into an act of mind, its independent objects, and their compresence. This is not the mere distinction of act from object, but is only understood as the distinction of an enjoying subject from a contemplated object, separate from it. This latter distinction is thus the more important. This initial proposition of realism is "naive" and incomplete. When further examined, it turns out to be a particular case of the compresence of interrelated reals cohering within a universe. Two consequences of the analysis may be stated. First, mind is a continuum of mental functions which are also brain functions of a certain degree of development, with the mental quality. Being mind or consciousness is a new empirical quality which emerges at a particular stage. The mind is thus located in the brain. Secondly, the alleged distinction of "contents" of sense from the "objects" of thought disappears. The difference is one of part and whole. In each case there is an object, and not a "content." Reasons were assigned to account for the contrary view. But Mr. Bosanquet has urged that the analysis fails, because a mind is a world, while its object is a fragment. If this were so, the analysis from which realism starts would be false from the beginning. But in fact the mind is as much a fragment as the object, and the object is in the same sense a world as the mind (and neither is). His further objection that the analysis fails to account for the riches of mind, its wealth of being, or for tertiary qualities like beauty, was examined, and it was shown (*a*) that the riches of mind are unaffected; they are but a complex of processes and tendencies, always compresent with their objects; and (*b*) that the reality, and the more perfect reality, which is mind, enters as a constituent into beauty. It was then shown that while objects are independent of the mind, the mind is in a certain sense dependent on objects, or rather implies them. But again, to suppose that this minimizes the self-existence of mind is to confuse independence with isolation. The very lateness of mind in the order of development is the condition of its perfection. But the most searching

objection to realism is that its objects are mere abstractions and dead; whereas it is urged they already imply mind, and things are thus continuous in kind with mind. Now, according to realism, objects have all the fundamental characters, of continuity, retention, and the like, which can be seen more easily and flagrantly in minds. Thus the objection confuses the specific characters of minds with the categorical, fundamental characters which are common to minds with things. It may indeed be said, metaphorically, that all finites are minds; but this is inexact; and at any rate it does not mean that things are "mind," but only that they are different ranks of empirical existences, called minds, because in a certain sense they "know," that is, are compresent with, one another. This led to an attempt to define the larger issues between realism and (absolute) idealism. For in the case of the latter, things are transformed in entering into the one, individual whole. But for realism, things in certain respects at least (intrinsic ones) remain in the whole what they are already. The whole is not the only reality, but the most complete, or perfect reality, in a second sense of perfection.—*Athenaeum.*

THE Experimental Psychologists will meet this year at Columbia University on Thursday, Friday, and Saturday, April 9 to 11. The scientific sessions will be preceded by a dinner on Wednesday evening, April 8, in honor of Professor James McKeen Cattell.

PROFESSOR THOMAS H. HAINES, of Ohio State University, who is on leave of absence, is conducting the courses in psychology at Smith College during the present semester.

DR. GEORGE R. M. WELLS, of Oberlin College, has been advanced to an associate professorship of psychology.

THE Western Philosophical Association will hold its annual meeting at the University of Chicago, Thursday and Friday, April 9 and 10, in connection with the Conference on Legal and Social Philosophy, which is to occur at Chicago, April 10 and 11. It is proposed to devote the session of Thursday afternoon to a discussion of the Neo-Realistic Doctrine of Relations. The President's address will be given Thursday evening. At the session Friday morning, Professor Fite will lead a discussion of the subject of Natural Rights. On Friday afternoon the Association will hold a joint session with the Conference on Legal and Social Philosophy. The special topic to be considered is Rule *versus* Discretion.

PROFESSOR GEORGE STUART FULLERTON closed his lectures as the first Columbia exchange professor at the University of Vienna on February 21. After the final lecture, the Dean of the Philosophical Faculty made an address in which he announced that Professor Fullerton had been nominated honorary professor by the faculty, and closed with these words: "I am glad to be able to inform you that yesterday the Emperor confirmed your nomination as honorary professor. You are hereby given the continued right to lecture at our University, and I may express the wish that you will frequently make use of it. In this spirit, let me say *auf Wiedersehen.*"